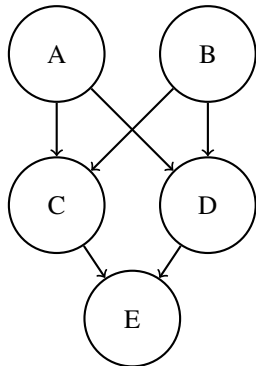


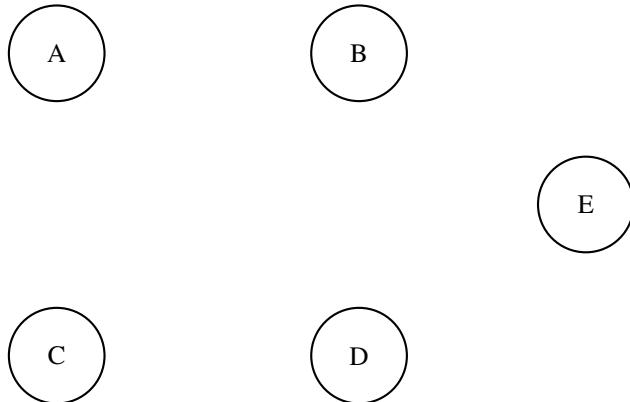
## Q1. Bayes Nets and Joint Distributions

- (a) Write down the joint probability distribution associated with the following Bayes Net. Express the answer as a product of terms representing individual conditional probabilities tables associated with this Bayes Net:



- (b) Draw the Bayes net associated with the following joint distribution:

$$P(A) \cdot P(B) \cdot P(C|A, B) \cdot P(D|C) \cdot P(E|B, C)$$



- (c) Do the following products of factors correspond to a valid joint distribution over the variables  $A, B, C, D$ ? (Circle FALSE or TRUE.)

(i)    FALSE    TRUE     $P(A) \cdot P(B) \cdot P(C|A) \cdot P(C|B) \cdot P(D|C)$

(ii)    FALSE    TRUE     $P(A) \cdot P(B|A) \cdot P(C) \cdot P(D|B, C)$

(iii)    FALSE    TRUE     $P(A) \cdot P(B|A) \cdot P(C) \cdot P(C|A) \cdot P(D)$

(iv)    FALSE    TRUE     $P(A|B) \cdot P(B|C) \cdot P(C|D) \cdot P(D|A)$

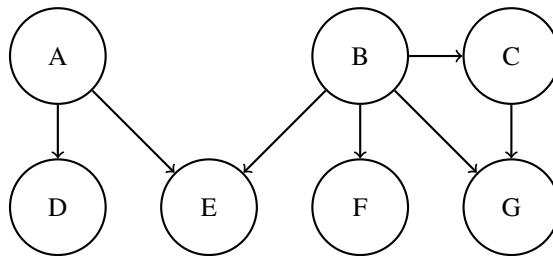
(d) What factor can be multiplied with the following factors to form a valid joint distribution? (Write “none” if the given set of factors can’t be turned into a joint by the inclusion of exactly one more factor.)

(i)  $P(A) \cdot P(B|A) \cdot P(C|A) \cdot P(E|B, C, D)$

(ii)  $P(D) \cdot P(B) \cdot P(C|D, B) \cdot P(E|C, D, A)$

(e) Answer the next questions based off of the Bayes Net below:

All variables have domains of  $\{-1, 0, 1\}$



(i) Before eliminating any variables or including any evidence, how many entries does the factor at G have?

(ii) Now we observe  $e = 1$  and want to query  $P(D|e = 1)$ , and you get to pick the first variable to be eliminated.

- Which choice would create the **largest** factor  $f_1$ ?

- Which choice would create the **smallest** factor  $f_1$ ?

## Q2. Probability and Bayes Nets

- (a) A, B, and C are random variables with binary domains. How many entries are in the following probability tables and what is the sum of the values in each table? Write a “?” in the box if there is not enough information given.

Table	Size	Sum
$P(A, B C)$		
$P(A +b,+c)$		
$P(+a B)$		

- (b) Circle true if the following probability equalities are valid and circle false if they are invalid (leave it blank if you don't wish to risk a guess). Each True/False question is worth 1 points. Leaving a question blank is worth 0 points. **Answering incorrectly is worth -1 points.**

No independence assumptions are made.

(i) [true or false]  $P(A, B) = P(A|B)P(A)$

(ii) [true or false]  $P(A|B)P(C|B) = P(A, C|B)$

(iii) [true or false]  $P(B, C) = \sum_{a \in A} P(B, C|A)$

(iv) [true or false]  $P(A, B, C, D) = P(C)P(D|C)P(A|C, D)P(B|A, C, D)$

- (c) Space Complexity of Bayes Nets

Consider a joint distribution over  $N$  variables. Let  $k$  be the domain size for all of these variables, and let  $d$  be the maximum indegree of any node in a Bayes net that encodes this distribution.

- (i) What is the space complexity of storing the entire joint distribution? Give an answer of the form  $O(\cdot)$ .
- (ii) Draw an example of a Bayes net over four binary variables such that it takes less space to store the Bayes net than to store the joint distribution.
- (iii) Draw an example of a Bayes net over four binary variables such that it takes more space to store the Bayes net than to store the joint distribution.