CS490/590: Lecture 17
Reinforcement Learning

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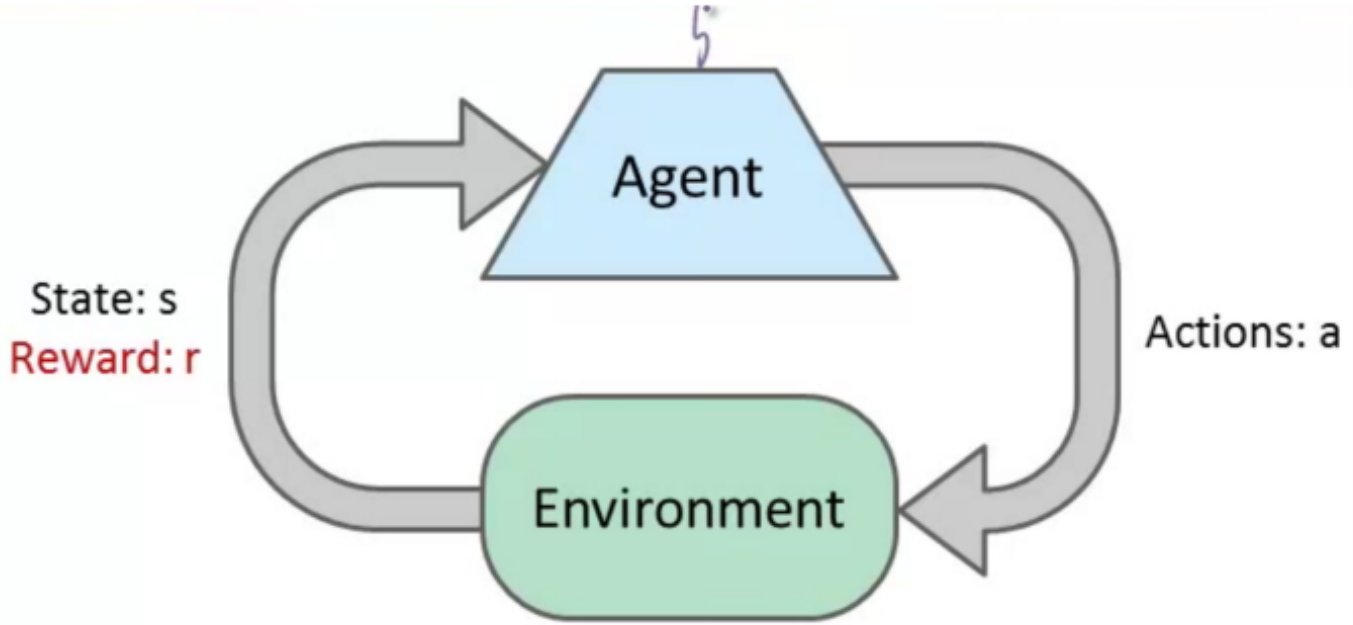
Adapted from Jimmy Ba and Bo Wang

Overview

- Agent interacts with an environment, which we treat as a black box
- Your RL code accesses it only through an API since it's external to the agent
  - I.e., you're not "allowed" to inspect the transition probabilities, reward distributions, etc.



Recap: Markov Decision Processes

- The environment is represented as a Markov decision process (MDP) $\mathcal{M}$.
- Markov assumption: all relevant information is encapsulated in the current state
- Components of an MDP:
  - initial state distribution $p(\mathrm{s}_0)$
  - transition distribution $p(\mathrm{s}_{t+1} \mid \mathrm{s}_t, \mathrm{a}_t)$
  - reward function $r(\mathrm{s}_t, \mathrm{a}_t)$
- policy $\pi_{\boldsymbol{\theta}}(\mathrm{a}_t \mid \mathrm{s}_t)$ parameterized by $\boldsymbol{\theta}$
- Assume a fully observable environment, i.e. s_t can be observed directly

Finite and Infinite Horizon

- Last time: finite horizon MDPs
  - Fixed number of steps T per episode
  - Maximize expected return $R = \mathbb{E}_{p(\tau)}[r(\tau)]$
- Now: more convenient to assume infinite horizon
  - We can't sum infinitely many rewards, so we need to discount them: \$100 a year from now is worth less than \$100 today
  - Discounted return

$$G_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots$$

  - Want to choose an action to maximize expected discounted return
  - The parameter $\gamma < 1$ is called the discount factor
    - small γ = myopic
    - large γ = farsighted

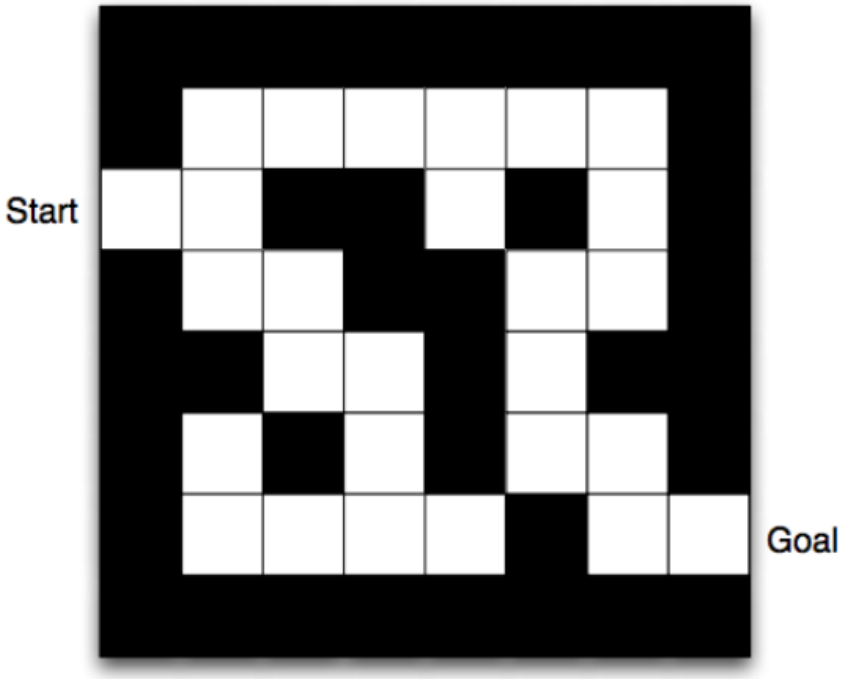
Value Function

- Value function $V^{\pi}(s)$ of a state s under policy π: the expected discounted return if we start in s and follow π

$$V^{\pi}(s) = \mathbb{E}[G_t \mid s_t = s]$$
$$= \mathbb{E}\left[\sum_{i=0}^{\infty} \gamma^i r_{t+i} \mid s_t = s\right]$$

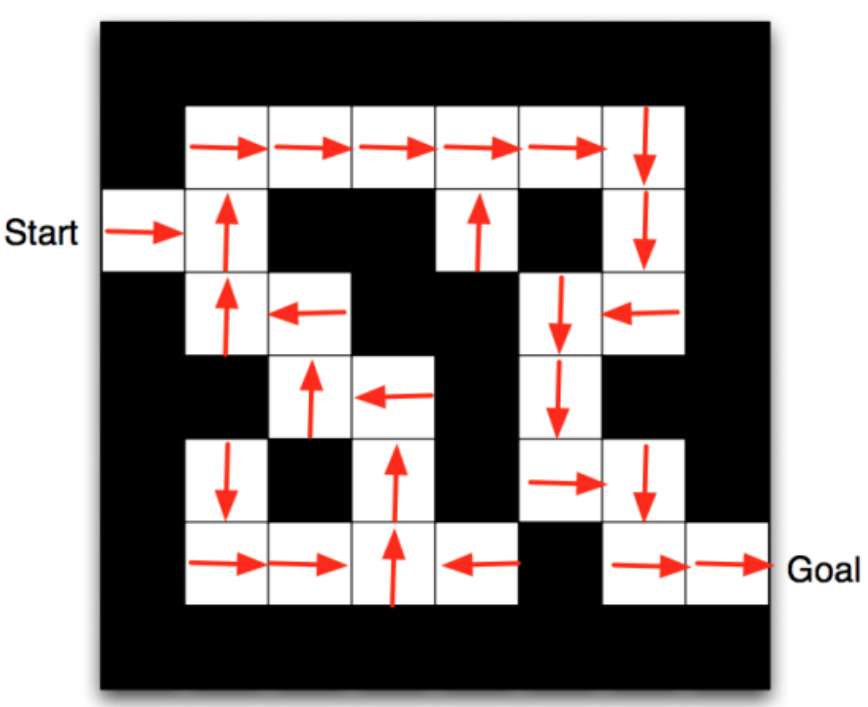
- Computing the value function is generally impractical, but we can try to approximate (learn) it
- The benefit is credit assignment: see directly how an action affects future returns rather than wait for rollouts

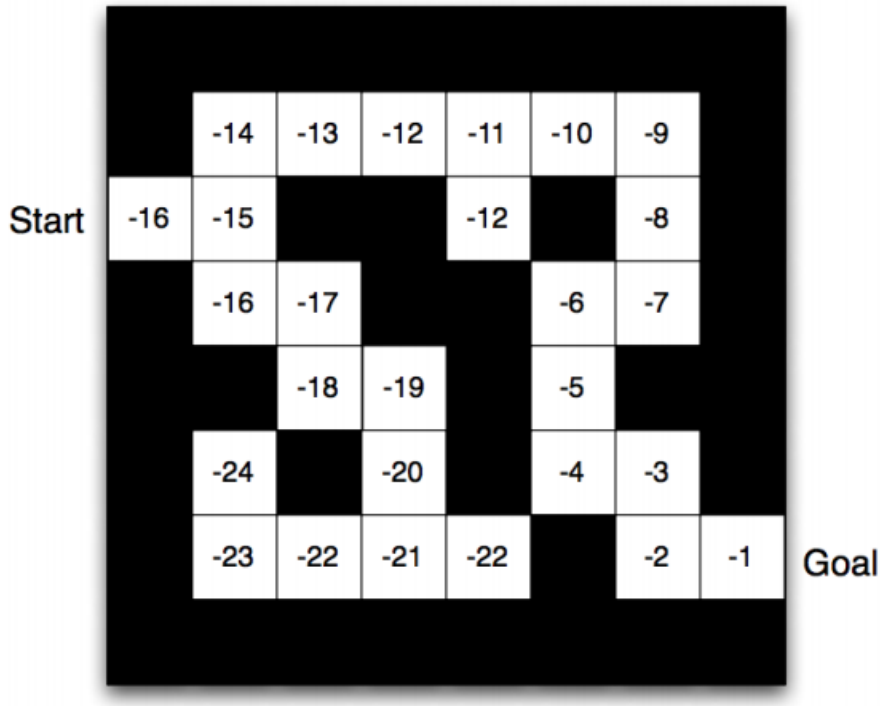
Value Function



- Rewards: -1 per time step
- Undiscounted ($\gamma = 1$)
- Actions: N, E, S, W
- State: current location

Value Function





Value Function

- The value function has a recursive formula

$$
\begin{aligned}
V^{\pi}(s) &= \mathop{\mathbb{E}}_{a_t, a_{t+i}, s_{t+i}} \left[\sum_{i=0}^{\infty} \gamma^i r_{t+i} | s_t = s \right] \\
&= \mathop{\mathbb{E}}_{a_t} [r_t | s_t = s] + \gamma \mathop{\mathbb{E}}_{a_t, a_{t+i}, s_{t+i}} \left[\sum_{i=1}^{\infty} \gamma^i r_{t+i+1} | s_t = s \right] \\
&= \mathop{\mathbb{E}}_{a_t} [r_t | s_t = s] + \gamma \mathop{\mathbb{E}}_{s_{t+1}} [V^{\pi}(s_{t+1}) | s_t = s] \\
&= \sum_{a,r} P^{\pi}(a|s_t) p(r|a, s_t) \cdot r + \gamma \sum_{a,s'} P^{\pi}(a|s_t) p(s'|a, s_t) \cdot V^{\pi}(s')
\end{aligned}
$$

Action-Value Function

- Can we use a value function to choose actions?

$$\arg\max_{a} r(s_t, a) + \gamma \mathbb{E}_{p(s_{t+1} \mid s_t, a_t)}[V^{\pi}(s_{t+1})]$$

Action-Value Function

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$$\arg\max_{a} r(s_t, a) + \gamma \mathbb{E}_{p(s_{t+1} \mid s_t, a_t)}[V^{\pi}(s_{t+1})]$$

- Problem: this requires taking the expectation with respect to the environment's dynamics, which we don't have direct access to!
- Instead learn an action-value function, or Q-function: expected returns if you take action a and then follow your policy

$$Q^{\pi}(s, a) = \mathbb{E}[G_t \mid s_t = s, a_t = a]$$

- Relationship:

$$V^{\pi}(s) = \sum_{a} \pi(a \mid s) Q^{\pi}(s, a)$$

- Optimal action:

$$\arg\max_{a} Q^{\pi}(s, a)$$

Bellman Equation

- The Bellman Equation is a recursive formula for the action-value function:

$$Q^{\pi}(s, a) = r(s, a) + \gamma \mathbb{E}_{p(s' \mid s,a)\, \pi(a' \mid s')}[Q^{\pi}(s', a')]$$

- There are various Bellman equations, and most RL algorithms are based on repeatedly applying one of them.

Optimal Bellman Equation

- The optimal policy π^* is the one that maximizes the expected discounted return, and the optimal action-value function Q^* is the action-value function for π^*.
- The Optimal Bellman Equation gives a recursive formula for Q^*:

$$Q^*(s, a) = r(s, a) + \gamma \mathbb{E}_{p(s' \mid s,a)} \left[\max_{a'} Q^*(s_{t+1}, a') \mid s_t = s, a_t = a \right]$$

- This system of equations characterizes the optimal action-value function. So maybe we can approximate Q^* by trying to solve the optimal Bellman equation!

Q-Learning

- Let Q be an action-value function which hopefully approximates Q^*.
- The Bellman error is the update to our expected return when we observe the next state s'.

$$\underbrace{r(s_t, a_t) + \gamma \max_a Q(s_{t+1}, a)}_{\text{inside } \mathbb{E} \text{ in RHS of Bellman eqn}} - Q(s_t, a_t)$$

- The Bellman equation says the Bellman error is 0 at convergence.
- Q-learning is an algorithm that repeatedly adjusts Q to minimize the Bellman error
- Each time we sample consecutive states and actions (s_t, a_t, s_{t+1}):

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \underbrace{\left[r(s_t, a_t) + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t) \right]}_{\text{Bellman error}}$$

Exploration-Exploitation Tradeoff

- Notice: Q-learning only learns about the states and actions it visits.
- Exploration-exploitation tradeoff: the agent should sometimes pick suboptimal actions in order to visit new states and actions.
- Simple solution: ϵ-greedy policy
  - With probability $1 - \epsilon$, choose the optimal action according to Q
  - With probability ϵ, choose a random action
- Believe it or not, ϵ-greedy is still used today!

Q-Learning

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Repeat (for each step of episode):
        Choose A from S using policy derived from Q (e.g., ε-greedy)
        Take action A, observe R, S'
        $Q(S,A) \leftarrow Q(S,A) + \alpha\left[R + \gamma \max_a Q(S',a) - Q(S,A)\right]$
        $S \leftarrow S'$;
    until S is terminal

Function Approximation

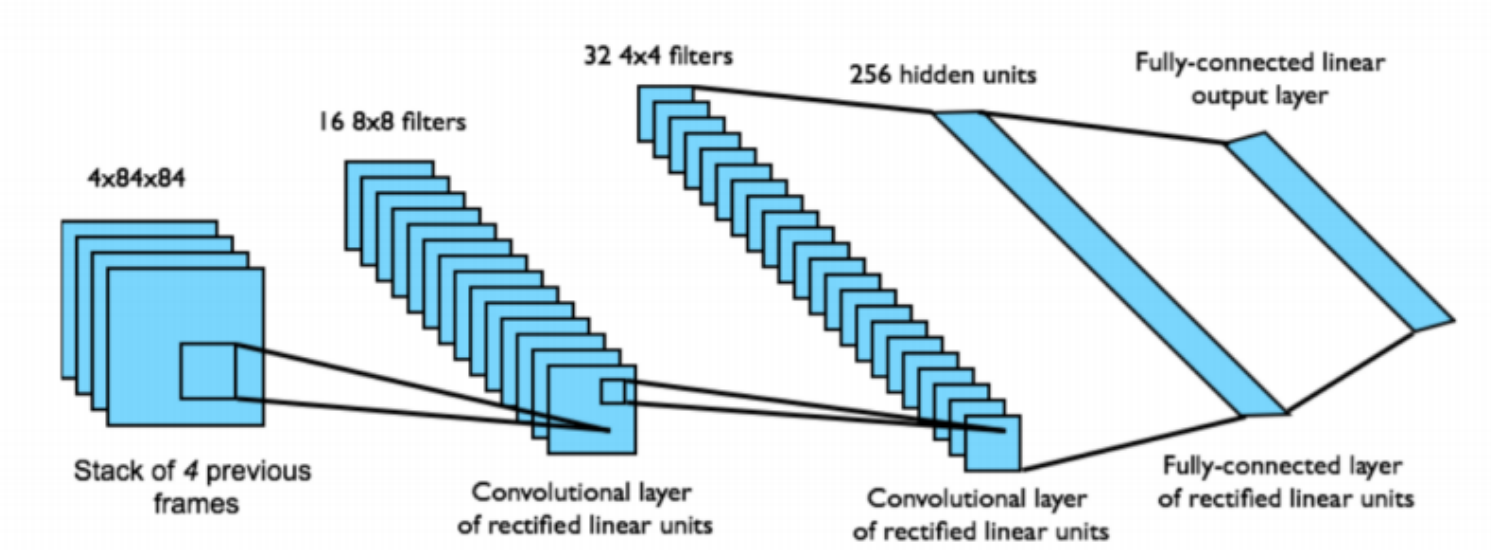
- So far, we've been assuming a tabular representation of Q: one entry for every state/action pair.
- This is impractical to store for all but the simplest problems, and doesn't share structure between related states.
- Solution: approximate Q using a parameterized function, e.g.
  - linear function approximation: $Q(\mathsf{s}, \mathsf{a}) = \mathsf{w}^\top \psi(\mathsf{s}, \mathsf{a})$
  - compute Q with a neural net
- Update Q using backprop:

$$t \leftarrow r(\mathsf{s}_t, \mathsf{a}_t) + \gamma \max_{\mathsf{a}} Q(\mathsf{s}_{t+1}, \mathsf{a})$$

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha(t - Q(\mathsf{s}, \mathsf{a}))\frac{\partial Q}{\partial \boldsymbol{\theta}}$$

Function Approximation with Neural Networks

- Approximating Q with a neural net is a decades-old idea, but DeepMind got it to work really well on Atari games in 2013 ("deep Q-learning")
- They used a very small network by today's standards



- Main technical innovation: store experience into a replay buffer, and perform Q-learning using stored experience
  - Gains sample efficiency by separating environment interaction from optimization — don't need new experience for every SGD update!

Atari

- Mnih et al., *Nature* 2015. Human-level control through deep reinforcement learning
- Network was given raw pixels as observations
- Same architecture shared between all games
- Assume fully observable environment, even though that's not the case
- After about a day of training on a particular game, often beat "human-level" performance (number of points within 5 minutes of play)
  - Did very well on reactive games, poorly on ones that require planning (e.g. Montezuma's Revenge)
- `https://www.youtube.com/watch?v=V1eYniJ0Rnk`
- `https://www.youtube.com/watch?v=4MlZncshy1Q`

Wireheading

- If rats have a lever that causes an electrode to stimulate certain "reward centers" in their brain, they'll keep pressing the lever at the expense of sleep, food, etc.
- RL algorithms show this "wireheading" behavior if the reward function isn't designed carefully
- `https://blog.openai.com/faulty-reward-functions/`

Policy Gradient vs. Q-Learning

- Policy gradient and Q-learning use two very different choices of representation: policies and value functions
- Advantage of both methods: don't need to model the environment
- Pros/cons of policy gradient
  - Pro: unbiased estimate of gradient of expected return
  - Pro: can handle a large space of actions (since you only need to sample one)
  - Con: high variance updates (implies poor sample efficiency)
  - Con: doesn't do credit assignment
- Pros/cons of Q-learning
  - Pro: lower variance updates, more sample efficient
  - Pro: does credit assignment
  - Con: biased updates since Q function is approximate (drinks its own Kool-Aid)
  - Con: hard to handle many actions (since you need to take the max)

AlphaGo

- Most of the problem domains we've discussed so far were natural application areas for deep learning (e.g. vision, language)
  - We know they can be done on a neural architecture (i.e. the human brain)
  - The predictions are inherently ambiguous, so we need to find statistical structure
- Board games are a classic AI domain which relied heavily on sophisticated search techniques with a little bit of machine learning
  - Full observations, deterministic environment — why would we need uncertainty?
- The second part of the lecture is about AlphaGo, DeepMind's Go playing system which took the world by storm in 2016 by defeating the human Go champion Lee Sedol
- Combines ideas from our last two lectures (policy gradient and value function learning)

AlphaGo

Some milestones in computer game playing:

- 1949 — Claude Shannon proposes the idea of game tree search, explaining how games could be solved algorithmically in principle
- 1951 — Alan Turing writes a chess program that he executes by hand
- 1956 — Arthur Samuel writes a program that plays checkers better than he does
- 1968 — An algorithm defeats human novices at Go

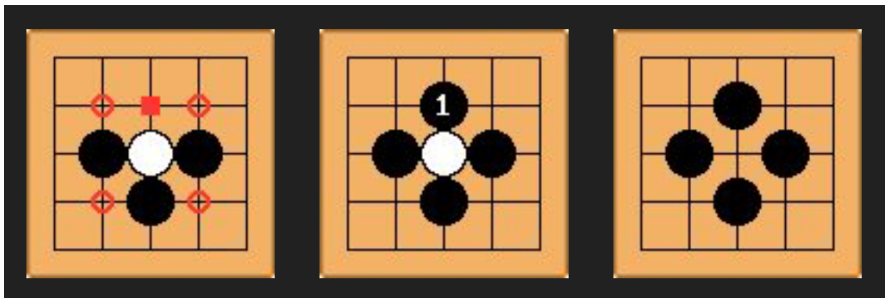
...silence...

- 1992 — TD-Gammon plays backgammon competitively with the best human players
- 1996 — Chinook wins the US National Checkers Championship
- 1997 — DeepBlue defeats world chess champion Garry Kasparov

After chess, Go was humanity's last stand

Go

- Played on a 19 $\times$ 19 board
- Two players, black and white, each place one stone per turn
- Capture opponent's stones by surrounding them



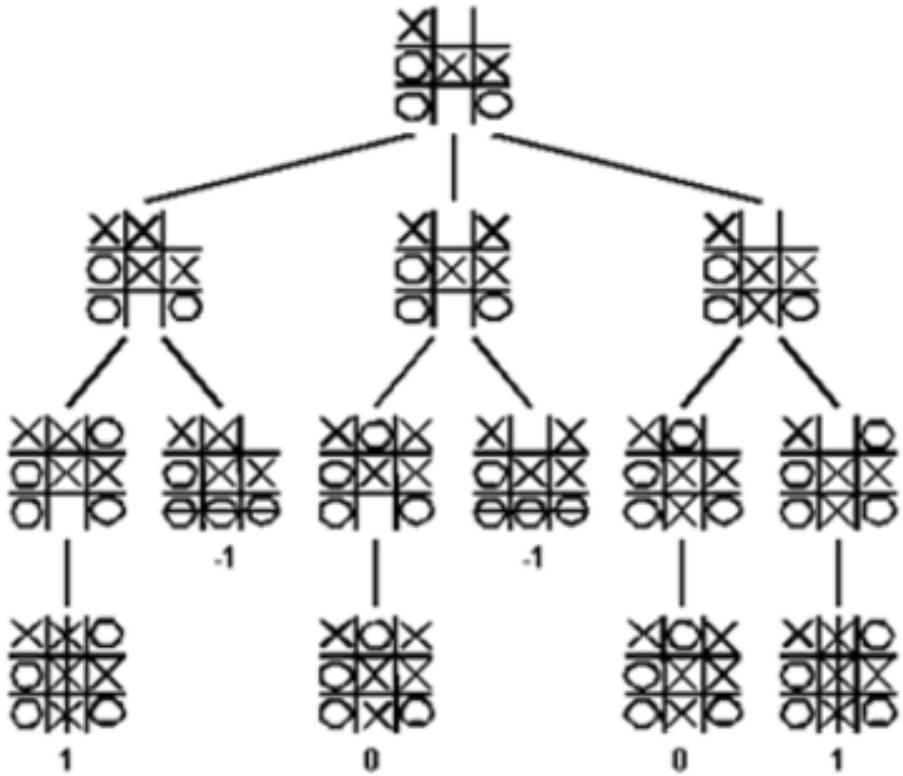
Go

What makes Go so challenging:

- Hundreds of legal moves from any position, many of which are plausible
- Games can last hundreds of moves
- Unlike Chess, endgames are too complicated to solve exactly (endgames had been a major strength of computer players for games like Chess)
- Heavily dependent on pattern recognition

Game Trees

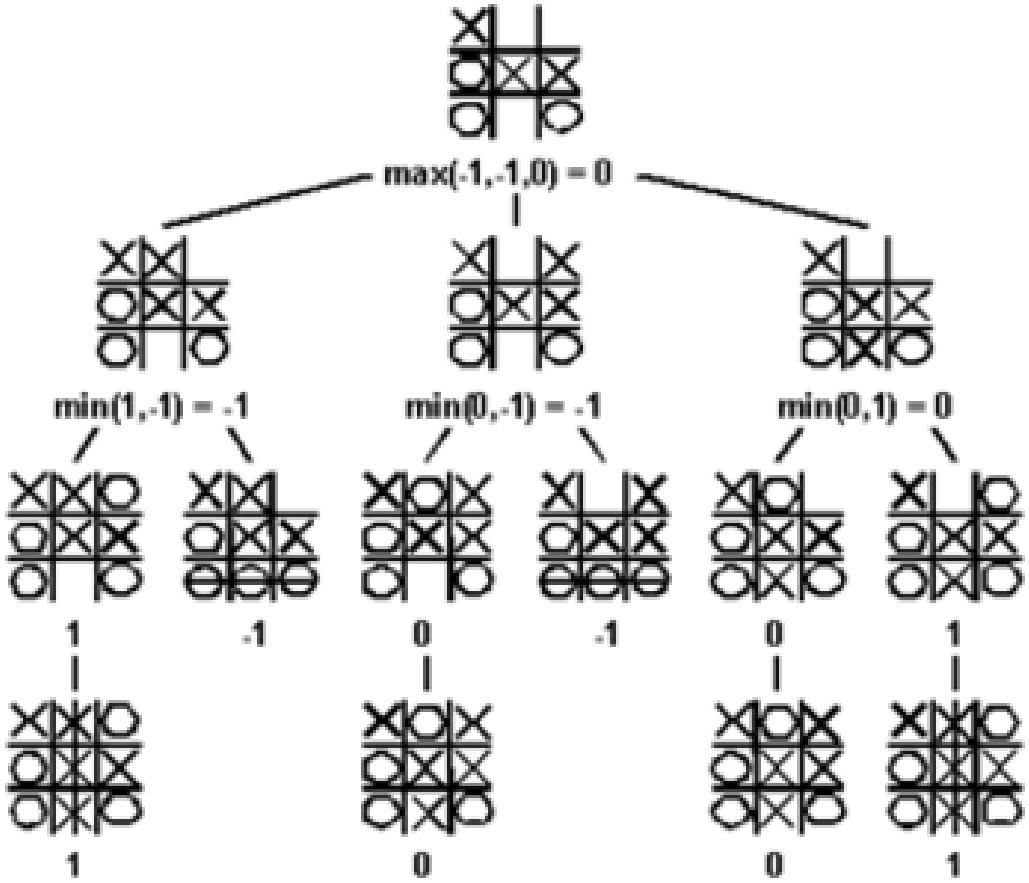
- Each node corresponds to a legal state of the game.
- The children of a node correspond to possible actions taken by a player.
- Leaf nodes are ones where we can compute the value since a win/draw condition was met



https://www.cs.cmu.edu/~adamchik/15-121/lectures/Game%20Trees/Game%20Trees.html

Game Trees

- To label the internal nodes, take the max over the children if it's Player 1's turn, min over the children if it's Player 2's turn



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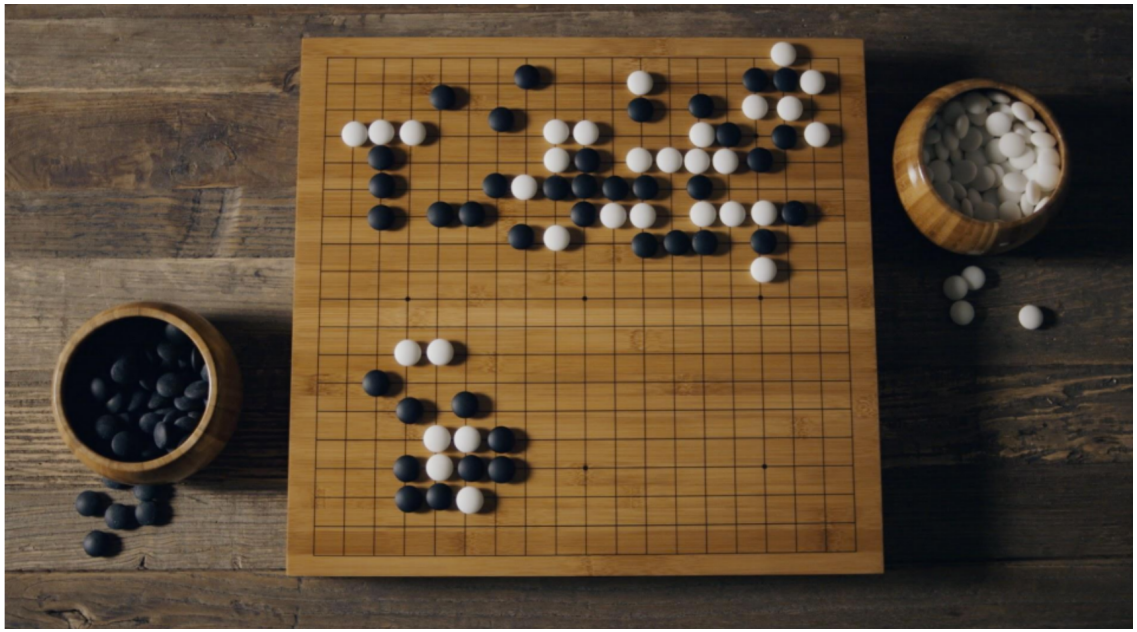
Now for DeepMind's computer Go player, AlphaGo...



- Naive game tree search is intractable:
  - The game tree is too deep.
  - The game tree is too wide.

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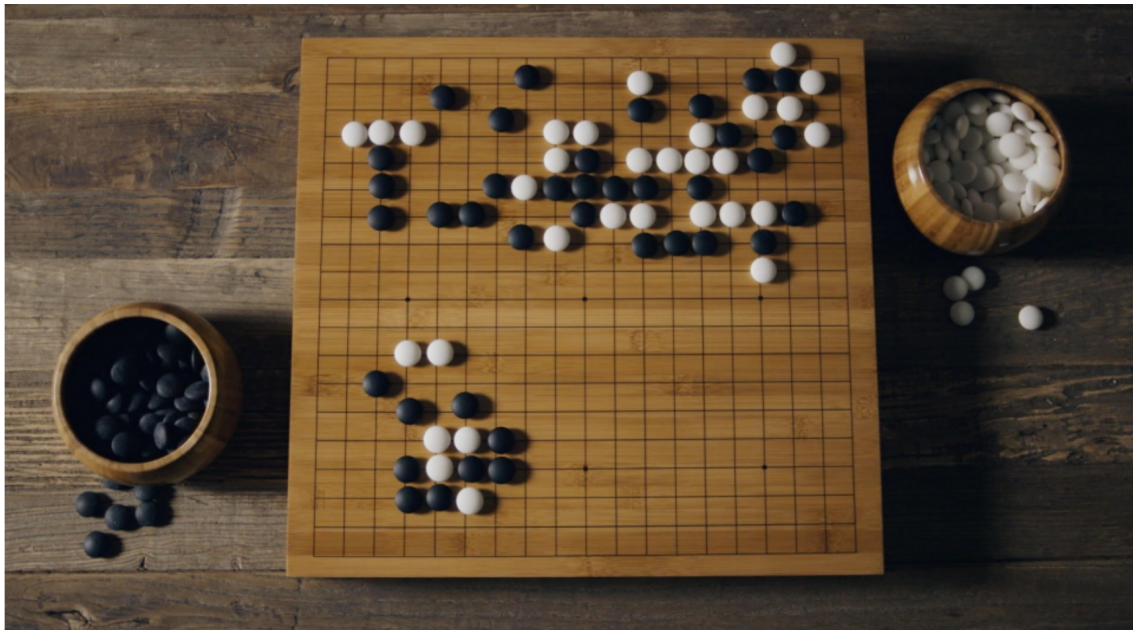
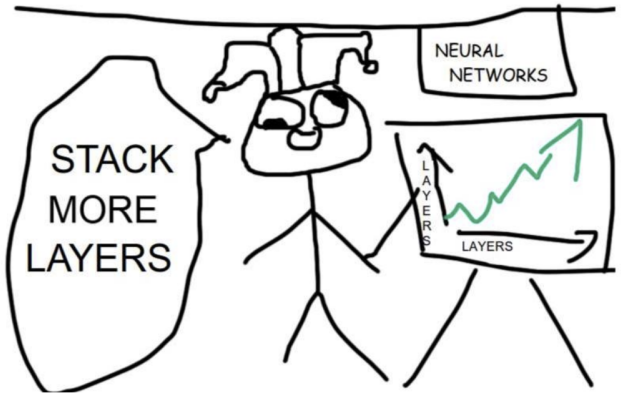


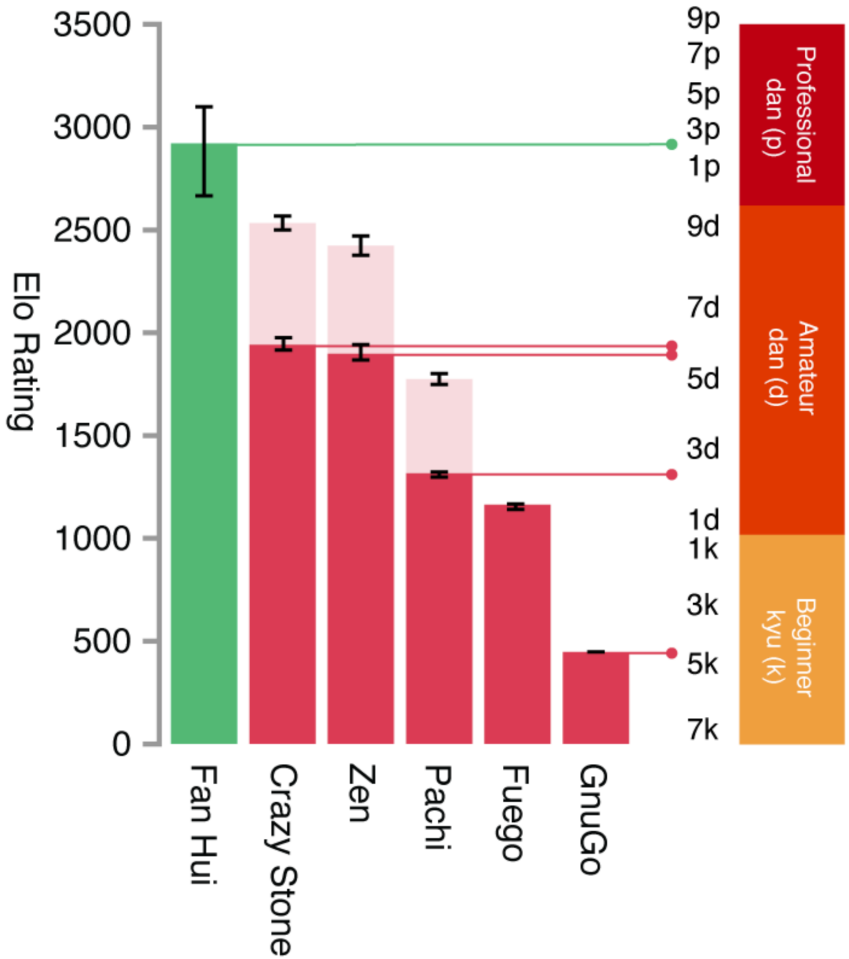


Supervised Learning to Predict Expert Moves

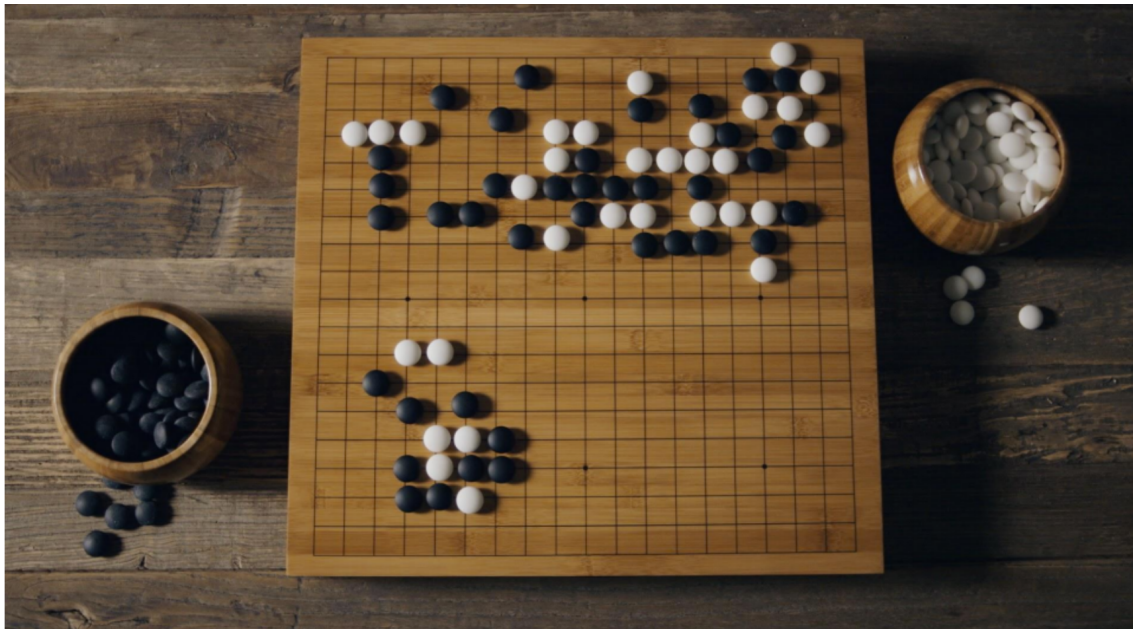
- Can a computer play Go without any search?
- **Input:** a 19×19 ternary (black/white/empty) image — about half the size of MNIST!
- **Prediction:** a distribution over all (legal) next moves
- **Training data:** KGS Go Server, consisting of 160,000 games and 29 million board/next-move pairs
- **Architecture:** fairly generic conv net
- When playing for real, choose the highest-probability move rather than sampling from the distribution
- This network, which just predicted expert moves, could beat a fairly strong program called GnuGo 97% of the time.
    - This was amazing — basically all strong game players had been based on some sort of search over the game tree

Supervised Learning to Predict Expert Moves

GnuGo is not good enough...



Now for DeepMind's computer Go player, AlphaGo...



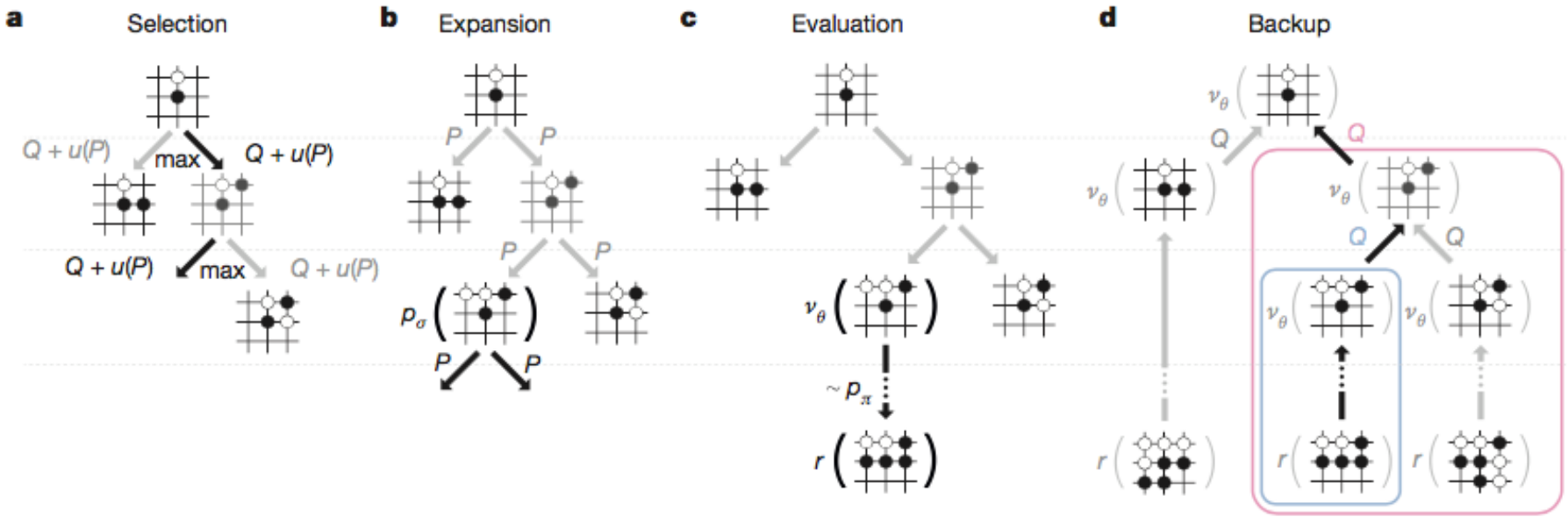
- Naive game tree search is intractable:
  - The game tree is too deep.
  - The game tree is too wide.

Approximate Evaluation

- As Claude Shannon pointed out in 1949, for games with finite numbers of states, you can solve them in principle by drawing out the whole game tree.
- Ways to deal with the exponential blowup
  - Search to some fixed depth, and then estimate the value using an evaluation function
  - Prioritize exploring the most promising actions for each player (according to the evaluation function)
- Having a good evaluation function is key to good performance
  - Traditionally, this was the main application of machine learning to game playing
  - For programs like Deep Blue, the evaluation function would be a learned linear function of carefully hand-designed features

Monte Carlo Tree Search

- In 2006, computer Go was revolutionized by a technique called Monte Carlo Tree Search.

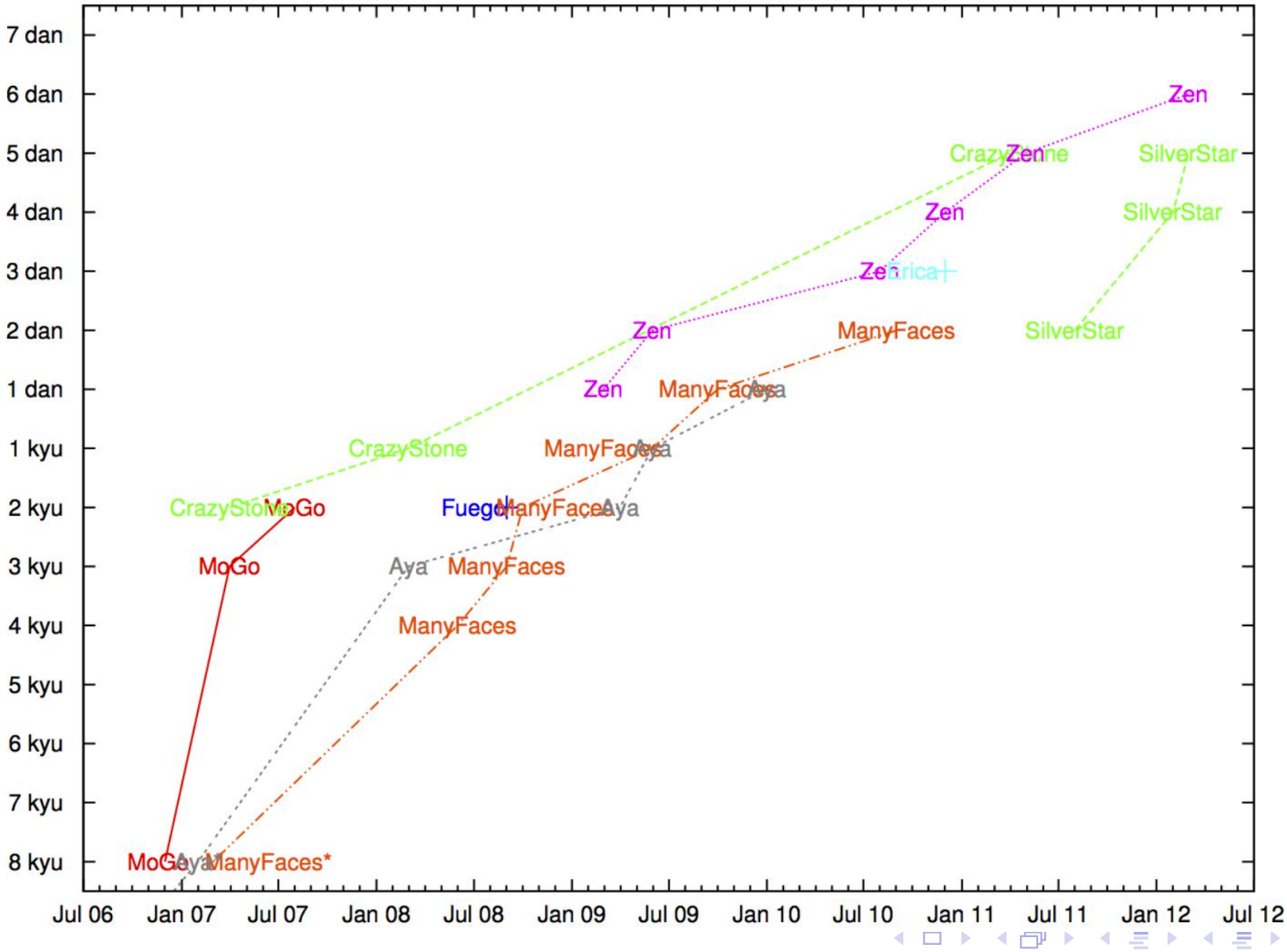


Silver et al., 2016

- Estimate the value of a position by simulating lots of rollouts, i.e. games played randomly using a quick-and-dirty policy
- Keep track of number of wins and losses for each node in the tree
- Key question: how to select which parts of the tree to evaluate?

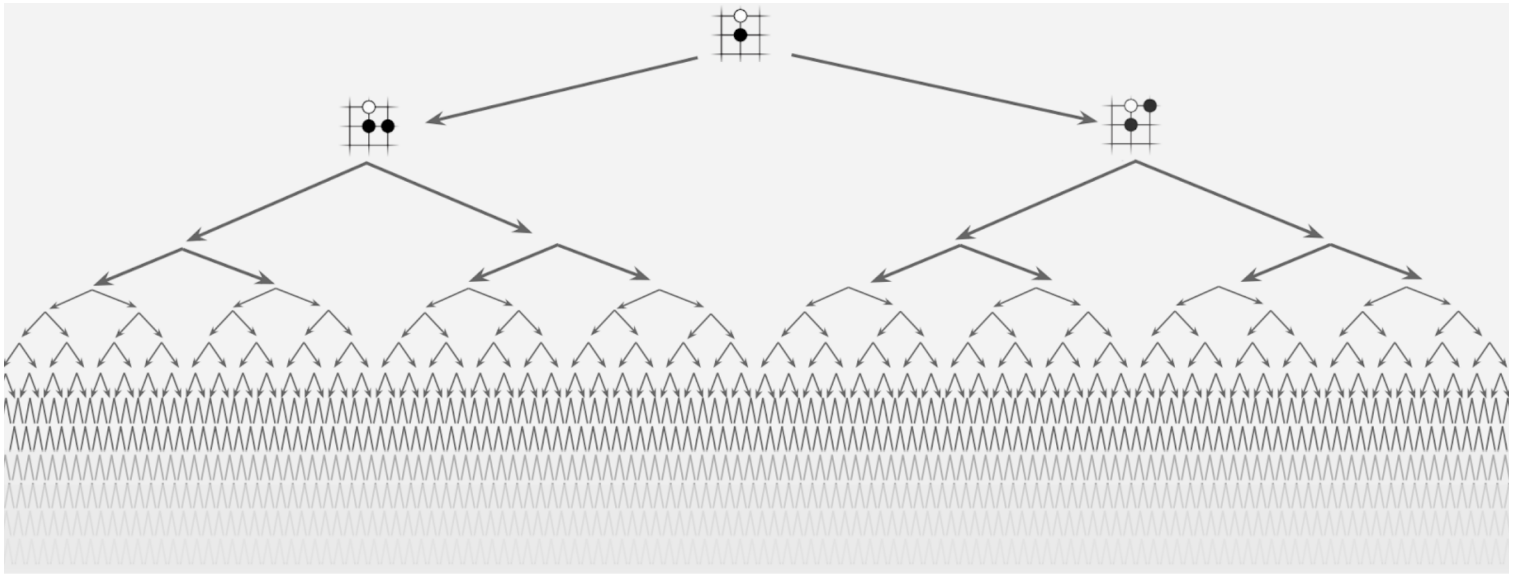
Monte Carlo Tree Search

Now for DeepMind's computer Go player, AlphaGo...



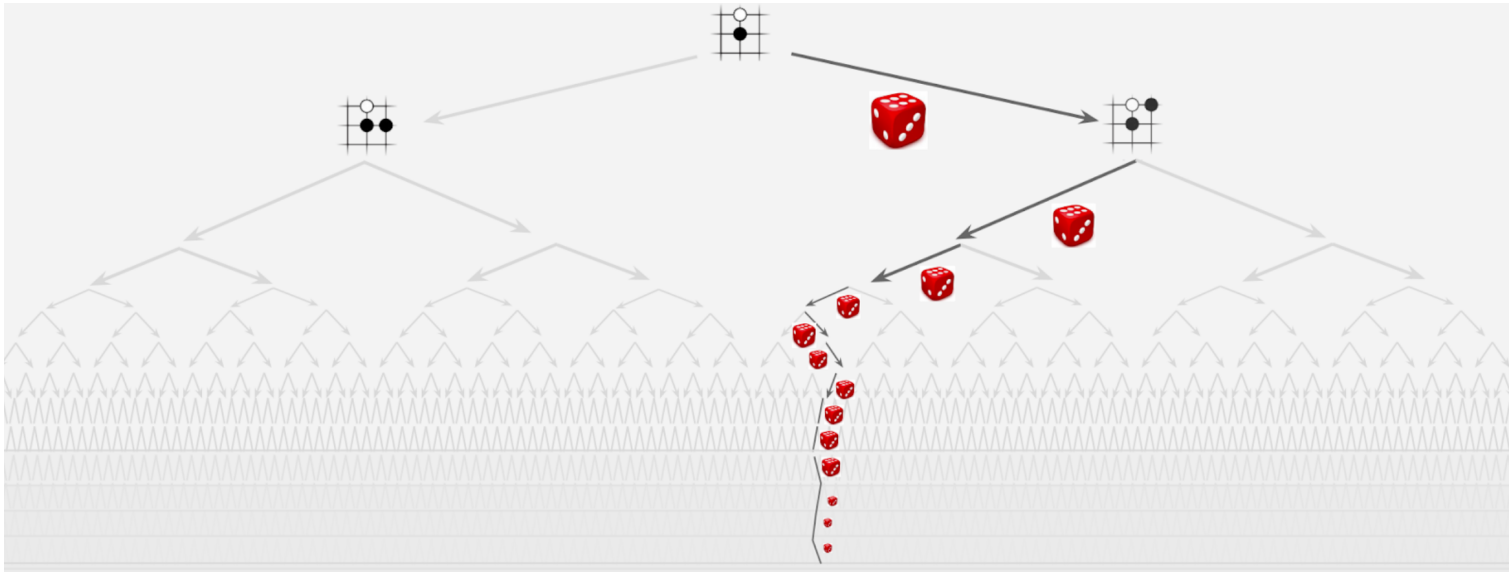
AlphaGo

Domain knowledge about the search tree:



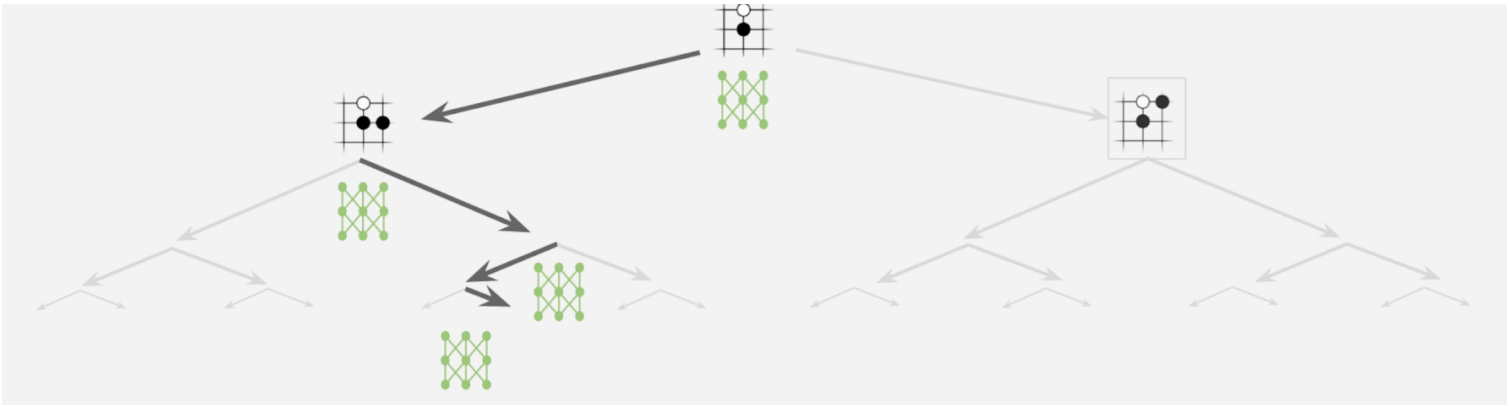
AlphaGo

Domain knowledge about the search tree: Monte Carlo Tree Search (MCTS)



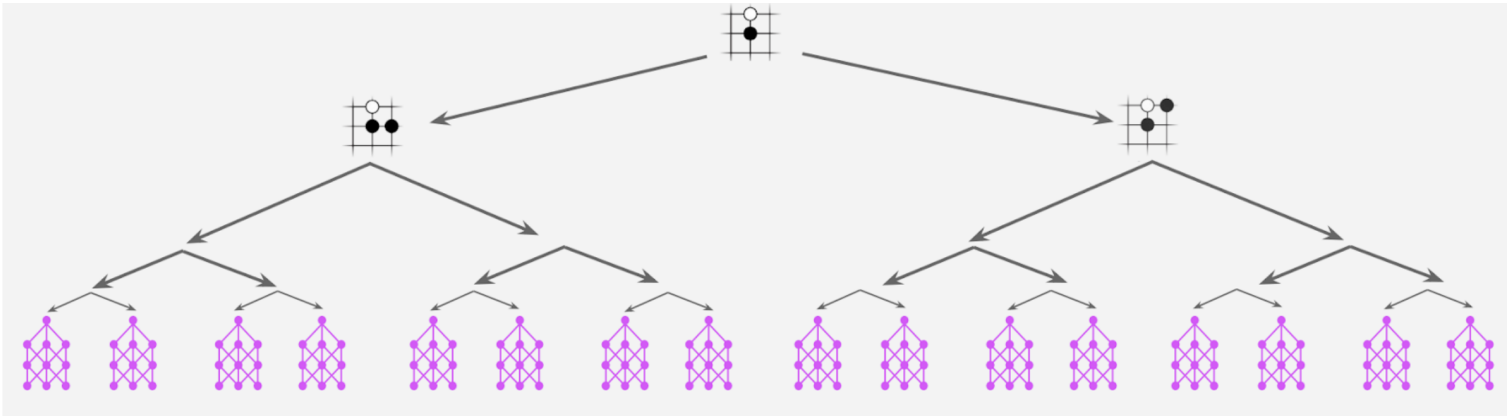
AlphaGo

Domain knowledge about the search tree: policy net



AlphaGo

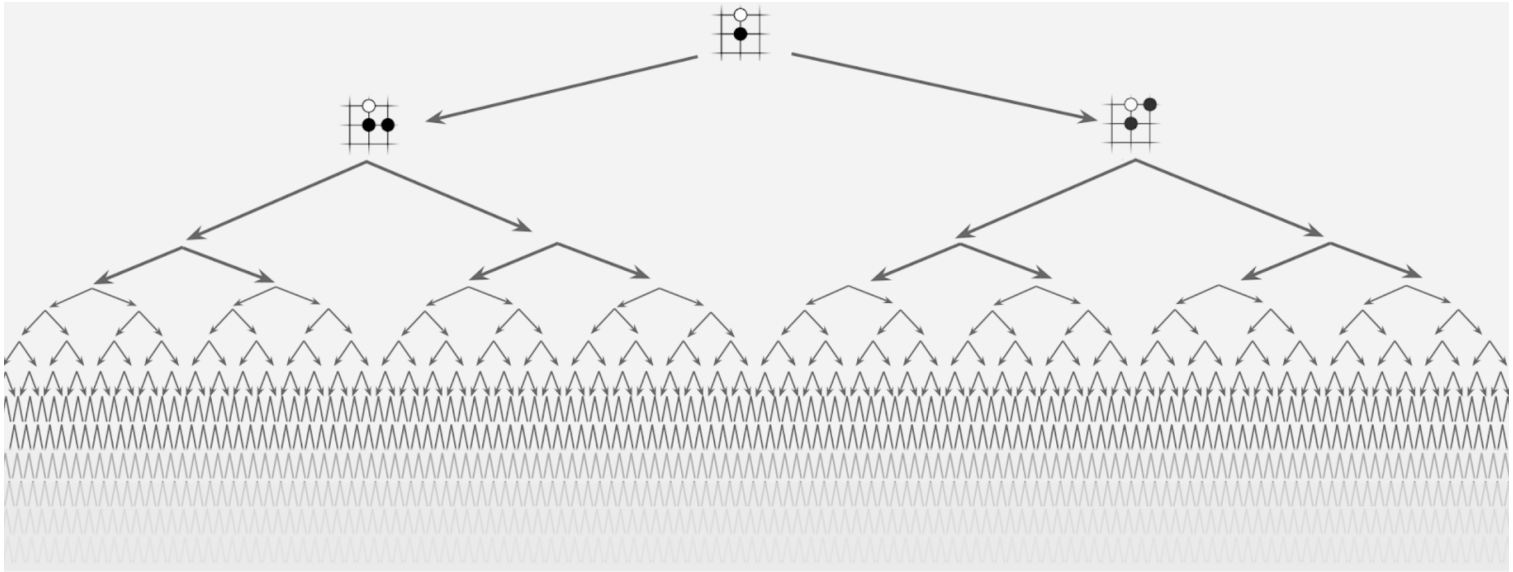
Domain knowledge about the search tree: value function



AlphaGo

Put it all together:

- MCTS + Policy: Reduce the search width.
- Value function: Reduce the search depth.



AlphaGo

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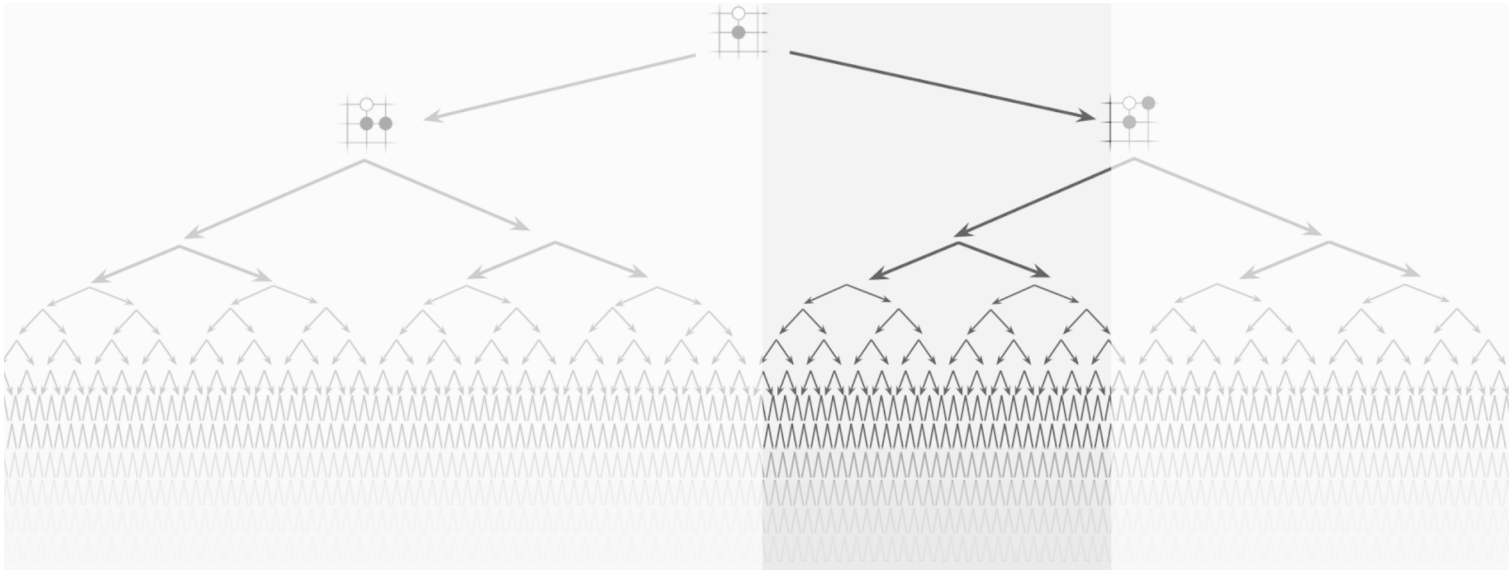
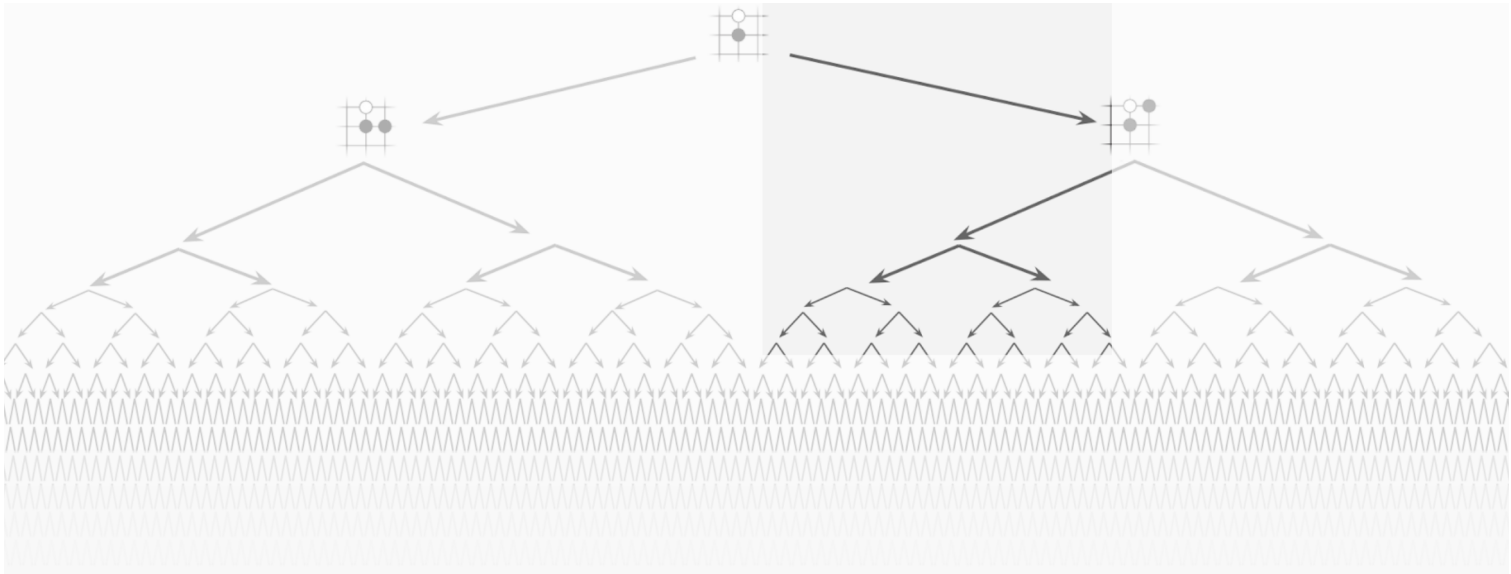


AlphaGo

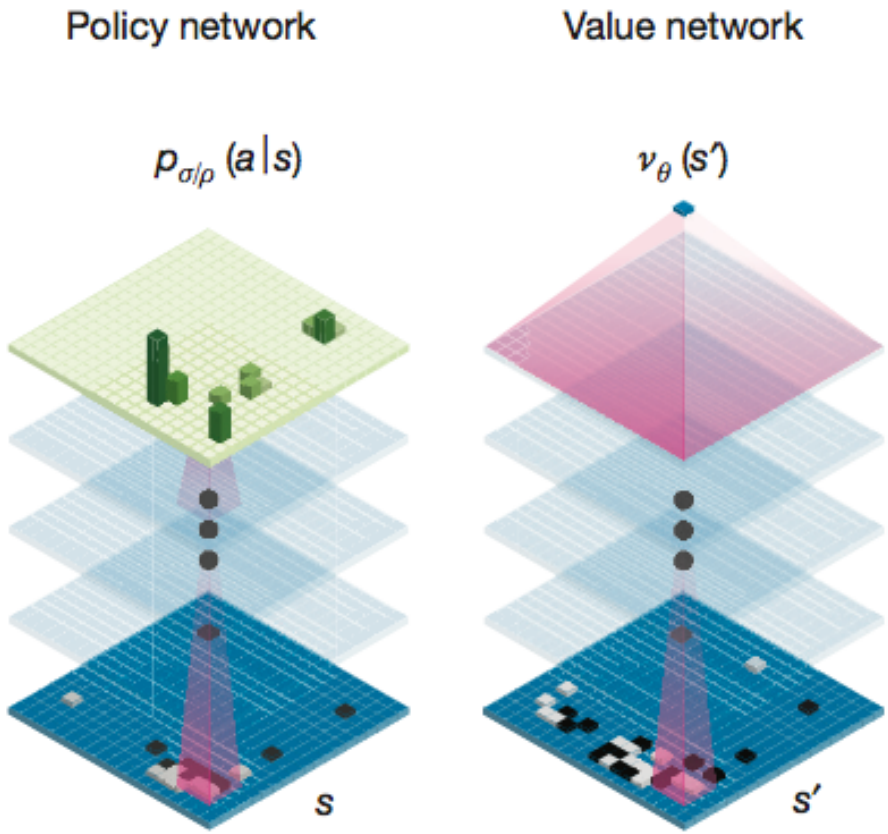
Put it all together:

- MCTS + Policy: Recude the search width.
- Value function: Recude the search depth.



Tree Search and Value Networks

- We just saw the policy network. But AlphaGo also has another network called a value network.
- This network tries to predict, for a given position, which player has the advantage.
- This is just a vanilla conv net trained with least-squares regression.
- Data comes from the board positions and outcomes encountered during self-play.



Silver et al., 2016

Policy and Value Networks

- AlphaGo combined the policy and value networks with Monte Carlo Tree Search
- Policy network used to simulate rollouts
- Value network used to evaluate leaf positions

Self-Play and REINFORCE

- The problem from training with expert data: there are only 160,000 games in the database. What if we overfit?
- There is effecitvely infinite data from self-play
  - Have the network repeatedly play against itself as its opponent
  - For stability, it should also play against older versions of itself
- Start with the policy which samples from the predictive distribution over expert moves
  - The network which computes the policy is called the policy network
- REINFORCE algorithm: update the policy to maximize the expected reward r at the end of the game (in this case, $r = +1$ for win, -1 for loss)
- If $\boldsymbol{\theta}$ denotes the parameters of the policy network, a_t is the action at time t, and s_t is the state of the board, and z the rollout of the rest of the game using the current policy

$$R = \mathbb{E}_{a_t \sim p_{\boldsymbol{\theta}}(a_t \mid s_t)}[\mathbb{E}[r(z) \mid s_t, a_t]]$$

Self-Play and REINFORCE

- Gradient of the expected reward:

$$\begin{aligned}
\frac{\partial R}{\partial \boldsymbol{\theta}} &= \frac{\partial R}{\partial \boldsymbol{\theta}} \mathbb{E}_{a_t \sim p_{\boldsymbol{\theta}}(a_t \mid s_t)}[\mathbb{E}[r(z) \mid s_t, a_t]] \\
&= \frac{\partial}{\partial \boldsymbol{\theta}} \sum_{a_t} \sum_{z} p_{\boldsymbol{\theta}}(a_t \mid s_t) p(z \mid s_t, a_t) R(z) \\
&= \sum_{a_t} \sum_{z} p(z) R(z) \frac{\partial}{\partial \boldsymbol{\theta}} p_{\boldsymbol{\theta}}(a_t \mid s_t) \\
&= \sum_{a_t} \sum_{z} p(z \mid s_t, a_t) R(z) p_{\boldsymbol{\theta}}(a_t \mid s_t) \frac{\partial}{\partial \boldsymbol{\theta}} \log p_{\boldsymbol{\theta}}(a_t \mid s_t) \\
&= \mathbb{E}_{p_{\boldsymbol{\theta}}(a_t \mid s_t)} \left[\mathbb{E}_{p(z \mid s_t, a_t)} \left[R(z) \frac{\partial}{\partial \boldsymbol{\theta}} \log p_{\boldsymbol{\theta}}(a_t \mid s_t) \right] \right]
\end{aligned}$$

- English translation: sample the action from the policy, then sample the rollout for the rest of the game.
  - If you win, update the parameters to make the action more likely. If you lose, update them to make it less likely.

AlphaGo Timeline

- **Summer 2014** — start of the project (internship project for UofT grad student Chris Maddison)
- **October 2015** — AlphaGo defeats European champion
  - First time a computer Go player defeated a human professional without handicap — previously believed to be a decade away
- **January 2016** — publication of Nature article "Mastering the game of Go with deep neural networks and tree search"
- **March 2016** — AlphaGo defeats gradmaster Lee Sedol
- **October 2017** — AlphaGo Zero far surpasses the original AlphaGo without training on any human data
- **December 2017** — it beats the best chess programs too, for good measure

AlphaGo

- Most of the Go world expected AlphaGo to lose 5-0 (even after it had beaten the European champion)
- It won the match 4-1
- Some of its moves seemed bizarre to human experts, but turned out to be really good
- Its one loss occurred when Lee Sedol played a move unlike anything in the training data

AlphaGo

Further reading:

- Silver et al., 2016. Mastering the game of Go with deep neural networks and tree search. *Nature* http://www.nature.com/nature/journal/v529/n7587/full/nature16961.html
- Scientific American: https://www.scientificamerican.com/article/how-the-computer-beat-the-go-master/
- Talk by the DeepMind CEO: https://www.youtube.com/watch?v=aiwQsa_7ZIQ&list=PLqYmG7hTraZCGIymT8wVVIXLWkKPNBoFN&index=8