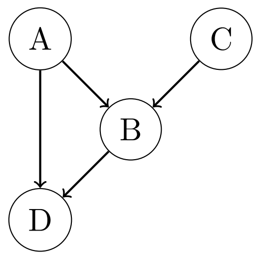
Introduction to Artificial Intelligence

Review Bayes Nets

Q1. Bayes Net Inference



Consider the Bayes net graph depicted above.

(a) **(i)** Select all conditional independences that are enforced by this Bayes net graph.

☐ $A \perp\!\!\!\perp B$ ☐ $A \perp\!\!\!\perp C \mid B$ ■ $D \perp\!\!\!\perp C \mid A, B$ ☐ $D \perp\!\!\!\perp C$

■ $A \perp\!\!\!\perp C$ ☐ $A \perp\!\!\!\perp C \mid D$ ☐ $A \perp\!\!\!\perp C \mid B, D$ ☐ $D \perp\!\!\!\perp C \mid B$

(ii) Because of these conditional independences, there are some distributions that cannot be represented by this Bayes net. What is the minimum set of edges that would need to be added such that the resulting Bayes net could represent any distribution?

☐ $A \to C$ ■ $C \to A$ ■ $C \to D$ ☐ $D \to C$

☐ $D \to A$ ☐ $D \to B$ ☐ $B \to C$ ☐ $B \to A$

Either ($C \to A$ AND $C \to D$) OR ($A \to C$ AND $C \to D$)

(b) For the rest of this Q2, we use the **original, unmodified** Bayes net depicted at the beginning of the problem statement. Here are some partially-filled conditional probability tables on A, B, C, and D. Note that these are not necessarily factors of the Bayes net. Fill in the six blank entries such that this distribution can be represented by the Bayes net.

A	C	$P(C \mid A)$
$+a$	$+c$	0.8
$+a$	$-c$	0.2
$-a$	$+c$	0.8
$-a$	$-c$	0.2

A	B	D	$P(D \mid A, B)$
$+a$	$+b$	$+d$	0.60
$+a$	$+b$	$-d$	0.40
$+a$	$-b$	$+d$	0.10
$+a$	$-b$	$-d$	0.90
$-a$	$+b$	$+d$	0.20
$-a$	$+b$	$-d$	0.80
$-a$	$-b$	$+d$	0.50
$-a$	$-b$	$-d$	0.50

A	B	C	$P(C \mid A, B)$
$+a$	$+b$	$+c$	0.50
$+a$	$+b$	$-c$	0.50
$+a$	$-b$	$+c$	0.20
$+a$	$-b$	$-c$	0.80
$-a$	$+b$	$+c$	0.90
$-a$	$+b$	$-c$	0.10
$-a$	$-b$	$+c$	0.40
$-a$	$-b$	$-c$	0.60

C	$P(C)$
$+c$	**(i)**
$-c$	**(ii)**

A	B	C	D	$P(D, C \mid A, B)$
$+a$	$+b$	$+c$	$+d$	**(iii)**
$+a$	$+b$	$-c$	$-d$	**(iv)**
$+a$	$-b$	$+c$	$+d$	**(v)**
$+a$	$-b$	$-c$	$-d$	**(vi)**
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

(i): 0.8

(ii): 0.2

(iii): $0.6 * 0.5 = 0.3$

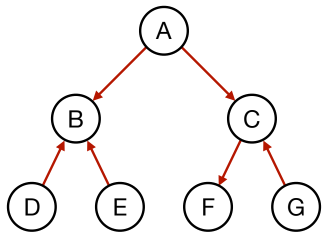
(iv): $0.4 * 0.5 = 0.2$

(v): $0.1 * 0.2 = 0.02$

(vi): $0.9 * 0.8 = 0.72$

Q2. Independence

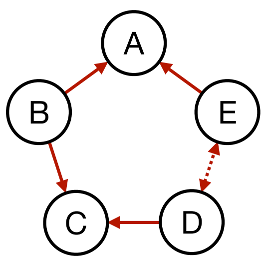
In each part of this question, you are given a Bayes' net where the edges do not have a direction. Assign a direction to every edge (by adding an arrowhead at one end of each edge) to ensure that the Bayes' Net structure implies the assumptions provided. You cannot add new edges. The Bayes' nets can imply more assumptions than listed, but they *must* imply the ones listed. There may be more than one correct solution.



(a)

Assumptions:

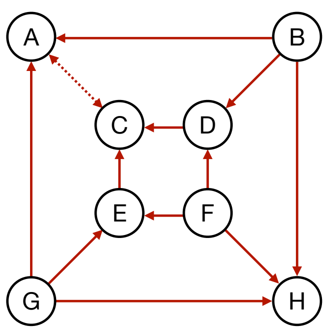
- $A \perp\!\!\!\perp G$
- $D \perp\!\!\!\perp E$
- $E \perp\!\!\!\perp F$
- $F \perp\!\!\!\perp G \mid C$



(b) (either direction is allowed for the edge DE)

Assumptions:

- $B \perp\!\!\!\perp E$
- $E \perp\!\!\!\perp C \mid D$



(c) (either direction is allowed for the edge AC)

Assumptions:

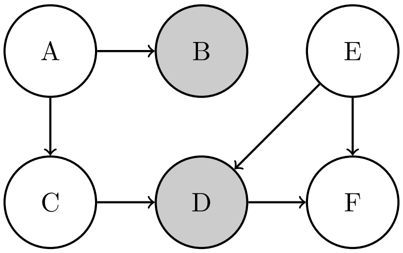
- $F \perp\!\!\!\perp G$
- $F \perp\!\!\!\perp B \mid G$
- $D \perp\!\!\!\perp E \mid F$

In order to have two nodes be independent, there must be an inactive triple along *all* paths between the two nodes.

1. $F \perp\!\!\!\perp G$, so the path FEG must have $F \rightarrow E \leftarrow G$
2. $F \perp\!\!\!\perp G$, so the path FHG must have $F \rightarrow H \leftarrow G$
3. $F \perp\!\!\!\perp B \mid G$, so the path FDB must have $F \rightarrow D \leftarrow B$ (we must later verify that G is not a descendant of D, but there is are no other edge directions along this path that will create an inactive triple)
4. $F \perp\!\!\!\perp B \mid G$, so the path FHB must have $F \rightarrow H \leftarrow B$ (we must later verify that G is not a descendant of H)
5. $D \perp\!\!\!\perp E \mid F$, so the path DCE must have $D \rightarrow C \leftarrow E$ (we must later verify that F is not a descendant of C)
6. $D \perp\!\!\!\perp E \mid F$, and we have already assigned some edge directions along path $DBAGE$. In particular, we have $D \leftarrow B - A - G \rightarrow E$. The only possible inactive triple we can create here is $B \rightarrow A \leftarrow G$ (we must later verify that F is not a descendant of A).
7. The only remaining edge to assign is $A - C$. We can assign either direction to this edge, and then verify that all required assumptions hold for the completed Bayes Net.

Q3. Bayes Nets and Sampling

You are given a bayes net with the following probability tables:



E	D	F	$P(F\|E,D)$
0	0	0	0.6
0	0	1	0.4
0	1	0	0.7
0	1	1	0.3
1	0	0	0.2
1	0	1	0.8
1	1	0	0.7
1	1	1	0.3

A	$P(A)$
0	0.75
1	0.25

A	B	$P(B\|A)$
0	0	0.1
0	1	0.9
1	0	0.5
1	1	0.5

A	C	$P(C\|A)$
0	0	0.3
0	1	0.7
1	0	0.7
1	1	0.3

E	$P(E)$
0	0.1
1	0.9

E	C	D	$P(D\|E,C)$
0	0	0	0.5
0	0	1	0.5
0	1	0	0.2
0	1	1	0.8
1	0	0	0.5
1	0	1	0.5
1	1	0	0.2
1	1	1	0.8

You want to know $P(C = 0|B = 1, D = 0)$ and decide to use sampling to approximate it.

(a) With prior sampling, what would be the likelihood of obtaining the sample [A=1, B=0, C=0, D=0, E=1, F=0]?

- ○ 0.25*0.1*0.3*0.9*0.8*0.7
- ● 0.25*0.5*0.7*0.5*0.9*0.2
- ○ 0.75*0.1*0.3*0.9*0.5*0.8
- ○ 0.25*0.5*0.3*0.2*0.9*0.2
- ○ 0.25*0.9*0.7*0.1*0.5*0.6
- ○ 0.75*0.1*0.3*0.9*0.5*0.2 + 0.25*0.5*0.7*0.5*0.9*0.2
- ○ Other ________ Prior sampling samples without taking the evidence into account, so the probability of the sample is P(A)P(B—A)P(C—A)P(D—C,E)P(E)P(F—E,D)

(b) Assume you obtained the sample [A = 1, B=1, C=0, D=0, E=1, F=1] through likelihood weighting. What is its weight?

- ○ 0.25*0.5*0.7*0.5*0.9*0.8
- ○ 0
- ○ 0.25*0.7*0.9*0.8 + 0.75*0.3*0.9*0.8
- ● 0.5*0.5
- ○ 0.25*0.5*0.7*0.5*0.8
- ○ 0.9*0.5 + 0.1*0.5
- ○ Other ________ The weight of a sample in gibbs sampling is the probability of the evidence given their parents: P(D=0—E=1,C=0)*P(B=1—A=1)

(c) You decide to use Gibb's sampling instead. Starting with the initialization [A = 1, B=1, C=0, D=0, E=0, F=0], suppose you resample F first, what is the probability that the next sample drawn is [A = 1, B=1, C=0, D=0, E=0, F=1]?

- ● 0.4
- ○ 0.6
- ○ 0.6*0.1*0.5
- ○ 0
- ○ 0.25*0.5*0.7*0.5*0.1*0.3
- ○ 0.9*0.5 + 0.1*0.5

○ Other ________ In Gibb's sampling, you resample individual vairables conditioned on the rest of the sample. The distribution of F given the rest of the sample is 0.4 for F=1 and 0.6 for F=0.